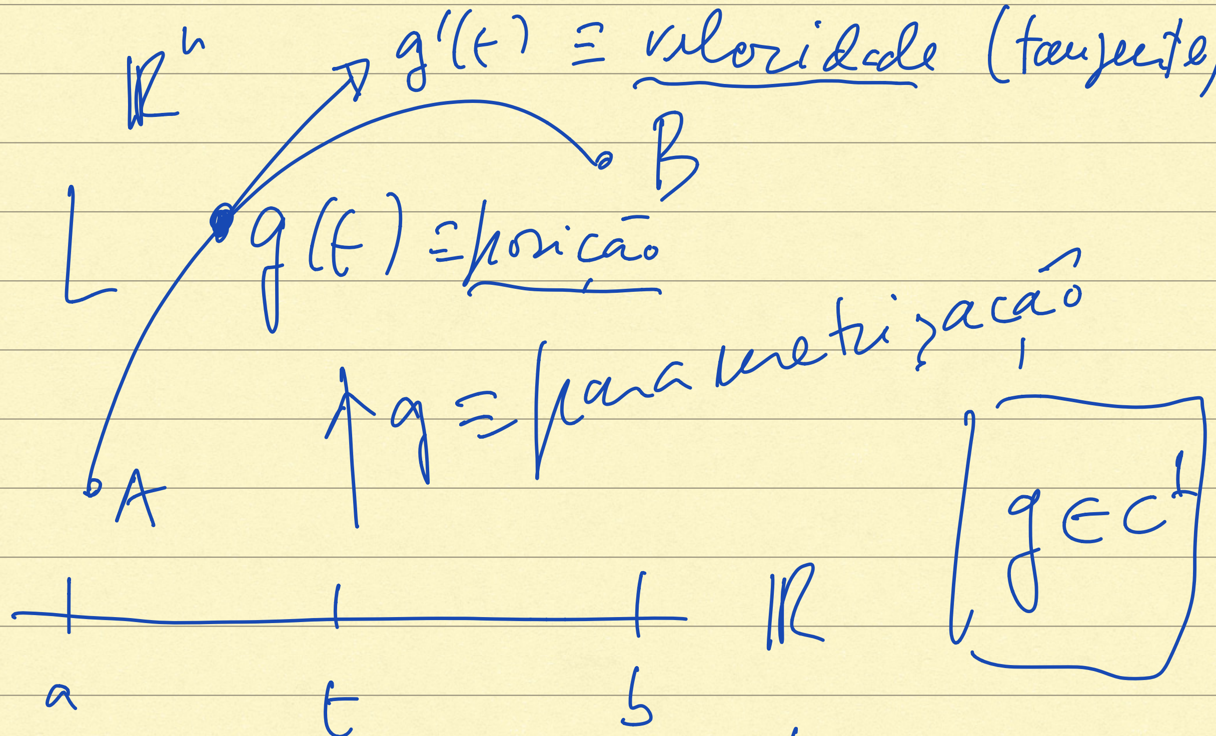
$$\boxed{\begin{array}{l}\text{Linhas — comprimento} \\ \text{Superfícies — Área}\end{array}}$$



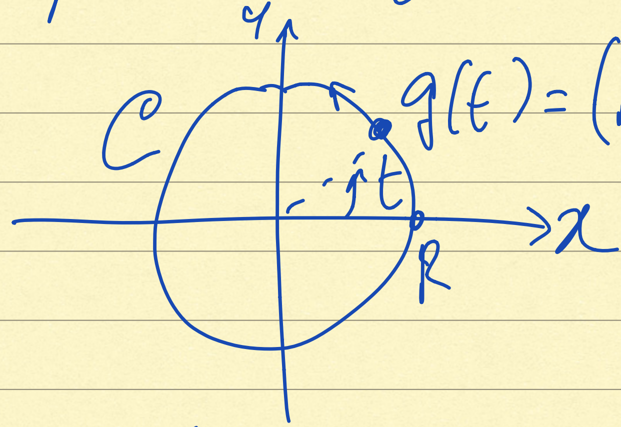
$$\ell(L) = \mathrm{vol}_1(L) = \int_a^b \underbrace{\|g'(t)\|}\, dt$$

<u>velocidade escalar</u> × tempo =

espaço percorrido

= comprimento da trajectória

<u>Exemplo:</u> $\mathcal{C} = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = R^2\}$



$g(t) = (R\cos t, R\,\text{sen}\, t)$ ✓

$0 \leq t \leq 2\pi$

$g : \mathbb{R} \to \mathbb{R}^2$

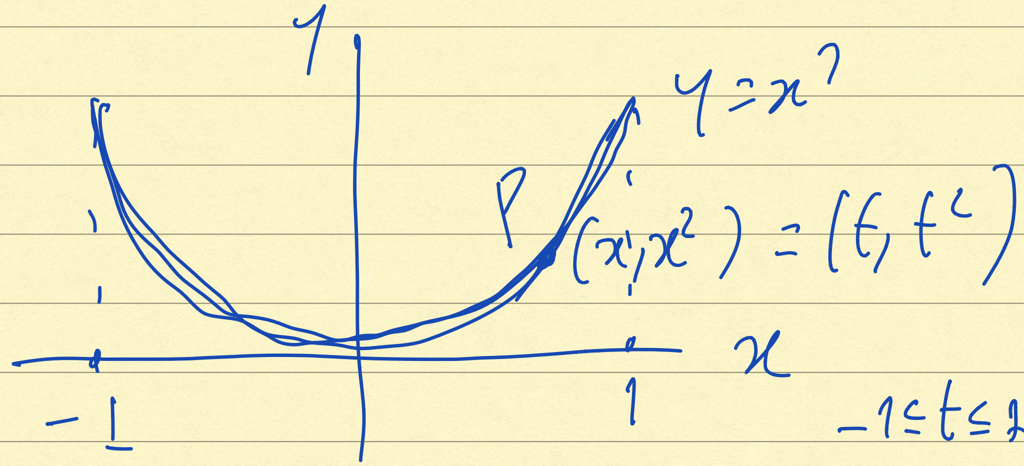
$g'(t) = (-R\,\text{sen}\, t, R\cos t)$ (vectorial)

$Dg(t) = \begin{bmatrix} -R\,\text{sen}\, t \\ R\cos t \end{bmatrix}$ (matricial)

$\|g'(t)\| = \sqrt{R^2\text{sen}^2 t + R^2\cos^2 t} = R.$

$\ell(\mathcal{C}) = \text{vol}_1(\mathcal{C}) = \int_0^{2\pi} R\,dt = 2\pi R$ ✓ //

Exemplo: $P = \{ (x,y) \in \mathbb{R}^2 : y = x^2 \leq 1 \}$



$-1 \leq t \leq 1$

$$g(t) = (t, t^2) \;;\; -1 \leq t \leq 1$$

$$\|g'(t)\| = \|(1, 2t)\| = \sqrt{1+4t^2}$$

$$\ell(P) = \int_{-1}^{1} \sqrt{1+4t^2}\, dt \quad (\text{difícil})$$

Exercício → funções hiperbólicas

$$sh(x) = \frac{e^x - e^{-x}}{2} \;,\; ch(x) = \frac{e^x + e^{-x}}{2}$$

$$ch^2(x) = sh^2(x) + 1.$$

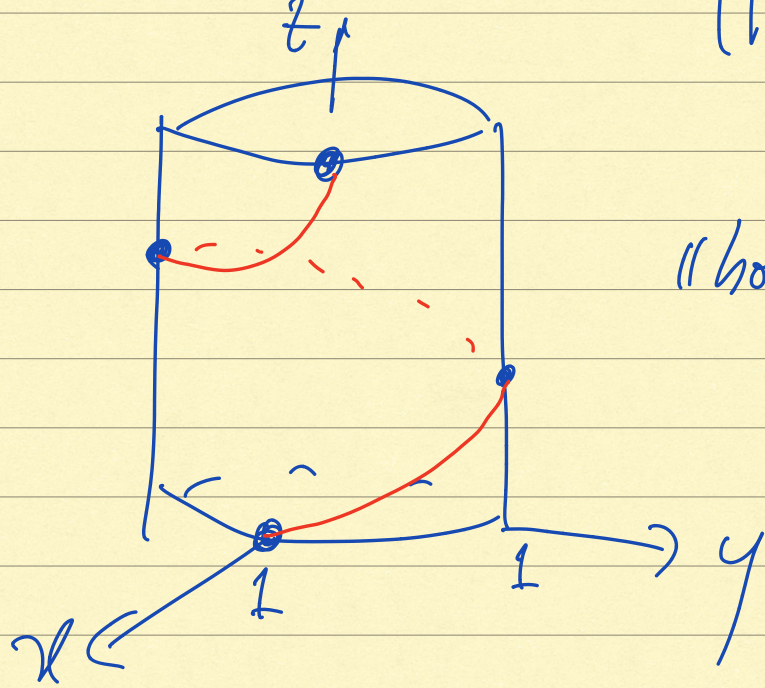
Exemplo: $L = \{(\cos t, \operatorname{sen} t, t)\} \subset \mathbb{R}^3$

$t \in [0, 2\pi]$.

$g(t) = (\cos t, \operatorname{sen} t, t)$

$= (x(t), y(t), \underbrace{z(t)})$

$\begin{cases} x^2 + y^2 = 1 \\ \boxed{z = t} \end{cases}$



"bobine"

$\|g'(t)\| = \|(-\operatorname{sen} t, \cos t, 1)\|$

$= \sqrt{\operatorname{sen}^2 t + \cos^2 t + 1} = \sqrt{2}.$

$$\ell(L) = \int_0^{2\pi} \sqrt{2}\, dt = 2\sqrt{2}\,\pi \,//$$

——— // ———

etc..

$\varphi : \mathbb{R}^n \to \mathbb{R}$, contínua.

$$\int_L \varphi = \int_a^b \varphi(g(t))\, \|g'(t)\|\, dt$$

1) $\varphi \equiv 1 \longrightarrow \int_L \varphi = \ell(L)$

2) $\varphi = \sigma$ (densidade de massa)

$\longrightarrow \int_L \varphi = M(L) \equiv$ Massa

etc.

$$g : \mathbb{R} \longrightarrow \mathbb{R}^n, \ C^1$$

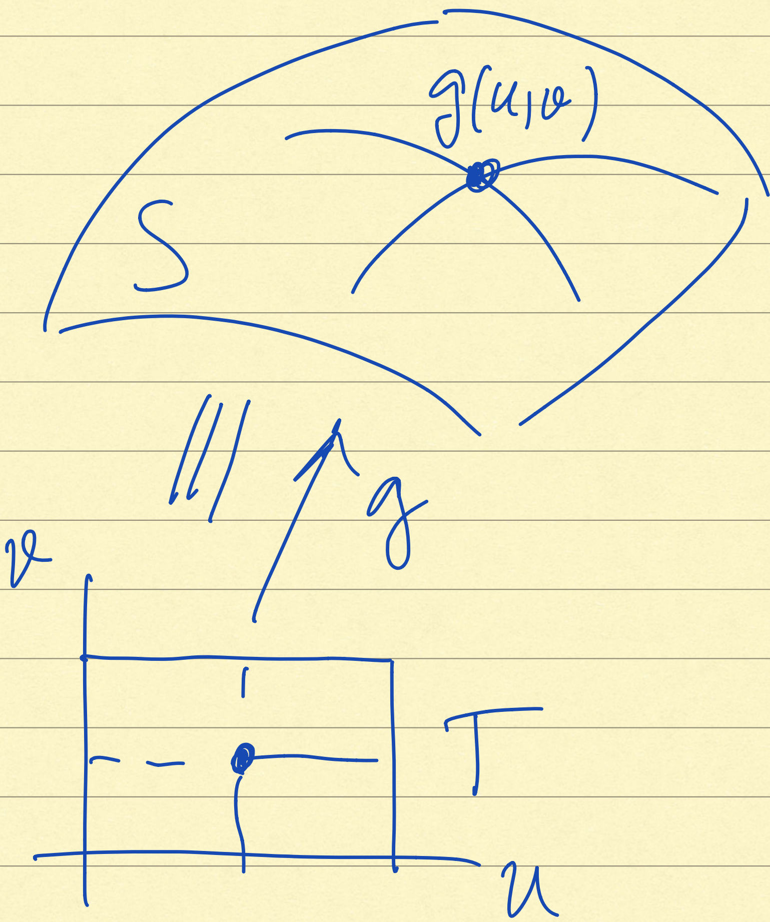
$$g'(t) \equiv Dg(t) = \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}_{n\times 1}$$

$$\boxed{\|g'(t)\| = \sqrt{\det Dg(t)^T \, Dg(t)}}$$

$$\underbrace{Dg^T Dg}_{1\times 1}$$

<u>Definição</u>: Área de uma superfície $S \subset \mathbb{R}^3$.

$$\dim(S) = 2$$



$F(x,y,z)=0$

$F:\mathbb{R}^3\to\mathbb{R}$, C^1

$DF(x,y,z)\neq(0,0,0)$

2 parâmetros

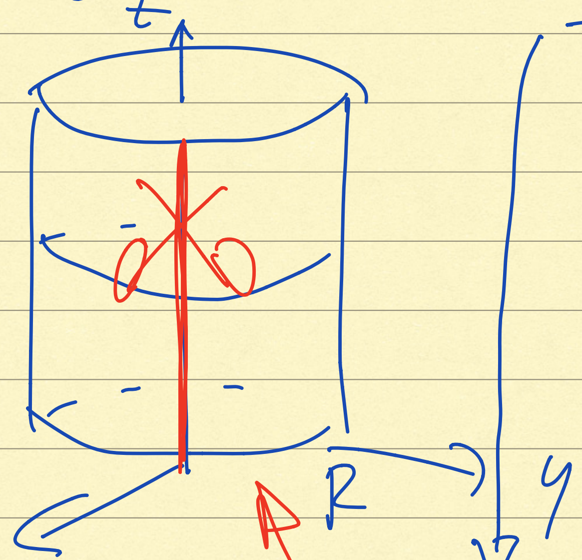
$S=g(T)$

$$\boxed{\mathrm{Vol}_2(S)=\iint_T \sqrt{\det Dg^T Dg}\; du\, dv}$$

Exemplo 1:

$$S = \{(x,y,z) \in \mathbb{R}^3 : \underline{x^2+y^2 = R^2}, \ 0 \le z \le h\}$$

S



1 eq.

$\Downarrow$

$\dim(S) = 2$

2 parâmetros

$\Downarrow$

(θ, z)

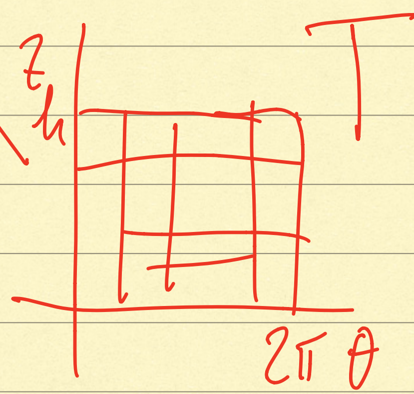
(ρ, θ, z)

$\rho = \sqrt{x^2+y^2}$

$$\boxed{\rho = R}$$

g

$$T : \begin{cases} 0 \le \theta \le 2\pi \\ 0 \le z \le h \end{cases}$$



$$g(\theta, z) = \big(x(\theta,z),\ y(\theta,z),\ z(\theta,z)\big)$$

$$= \big(R\cos\theta,\ \underline{R\sin\theta},\ \underline{z}\big)$$

$T: 0 \le \theta \le 2\pi \quad ; \quad 0 \le z \le h$

$$g: T \to \mathbb{R}^3, \quad T \subset \mathbb{R}^2$$

$$Dg(\theta, z) = \begin{bmatrix} -R\sin\theta & 0 \\ R\cos\theta & 0 \\ 0 & 1 \end{bmatrix}_{3\times 2}$$

$$\begin{array}{cc} D_\theta g & D_z g \end{array}$$

$$Dg(\theta,z)^T = \begin{array}{c} \begin{bmatrix} -R\sin\theta & R\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}_{2\times 3} \end{array} \begin{array}{l} D_\theta g \\ D_z g \end{array}$$

$$Dg^T Dg = \begin{bmatrix} R^2 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$$\det Dg^T Dg = \det \begin{bmatrix} R^2 & 0 \\ 0 & 1 \end{bmatrix} = R^2$$

$$\sqrt{\det Dg^T Dg} = R \,//.$$

$$\mathrm{Vol}_2(S) = \iint_T R \, d\theta \, dz$$

$$= \int_0^{2\pi} \left(\int_0^h R \, dz \right) d\theta = \underline{2\pi R} \, \underline{h}$$

Em geral:

$$Dg(u,v) = \begin{bmatrix} \vdots & \vdots \\ D_u g & D_v g \\ \vdots & \vdots \end{bmatrix}$$

$$Dg^T Dg = \underbrace{\begin{bmatrix} \cdots & D_u g & \cdots \\ \cdots & D_v g & \cdots \end{bmatrix}}_{2\times 3} \underbrace{\begin{bmatrix} \vdots & \vdots \\ D_u g & D_v g \\ \vdots & \vdots \end{bmatrix}}_{3\times 2}$$

$$\boxed{= \underbrace{\begin{bmatrix} \|D_u g\|^2 & D_u g \cdot D_v g \\ D_v g \cdot D_u g & \|D_v g\|^2 \end{bmatrix}}_{2\times 2}}$$

$$\int_S \varphi = \iint_T \varphi\big(g(u,v)\big) \sqrt{\det Dg^T Dg}\, du\, dv$$